Entangled Nets from Surface Drawings

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Overview

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- Is there a meaningful and simple way to combine the above approaches?

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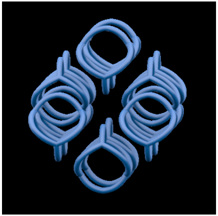

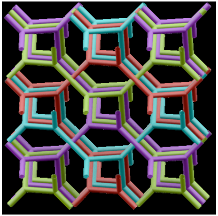

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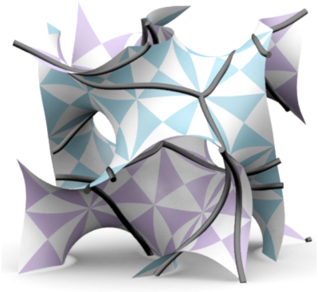

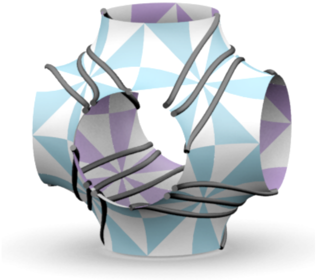

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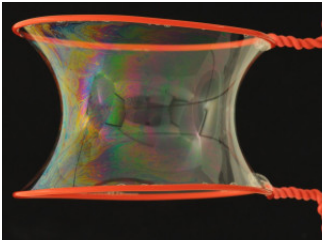

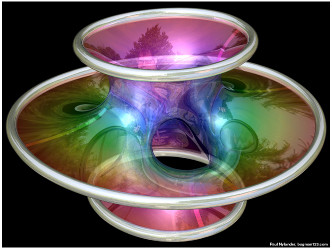

Figure: Minimal surfaces as soap films between wires (Paul Nylander)

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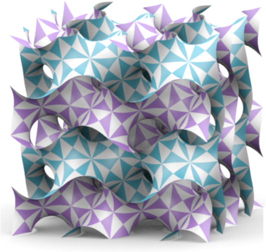

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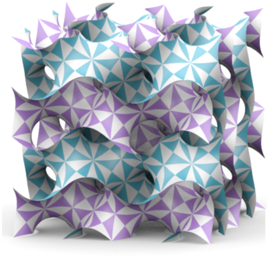

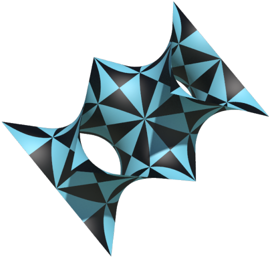

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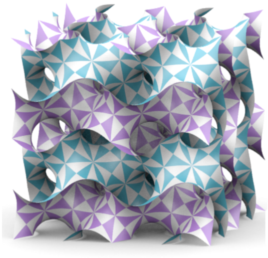

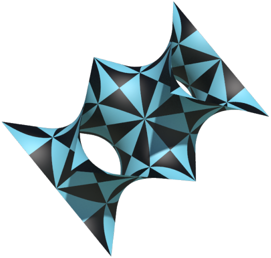

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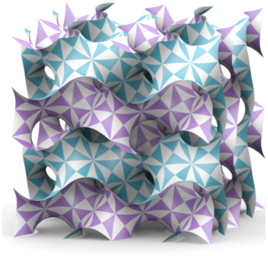

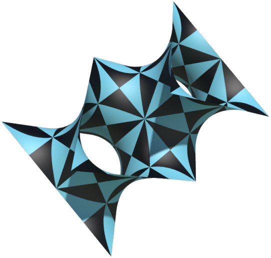

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- These symmetries yield the structure of a *hyperbolic orbifold*.

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- Prominent (triply periodic) minimal surfaces exhibit a high degree of symmetry
- They are covered by the hyperbolic plane $\mathbb{H}^2$

Orbifolds - Quick and Dirty

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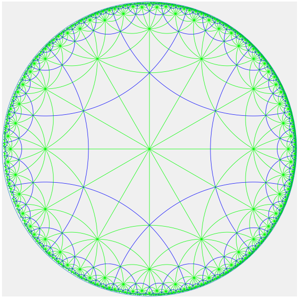


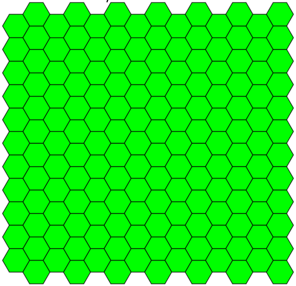
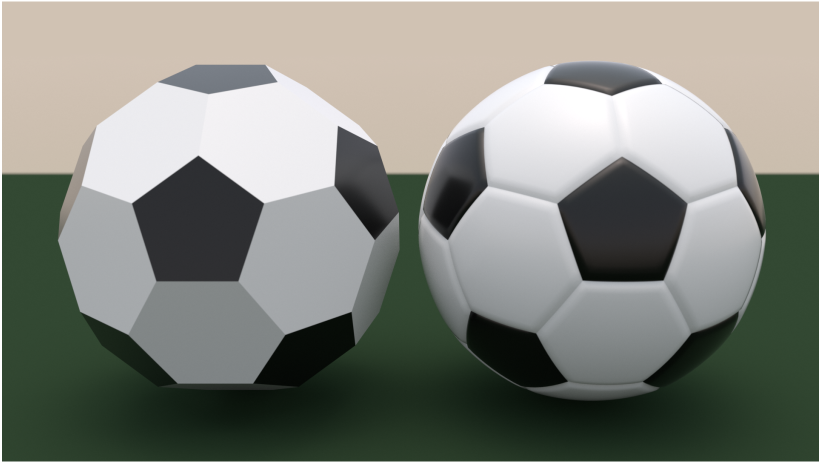


Figure: Euclidean and Hyperbolic 2D Developable Orbifolds

Examples



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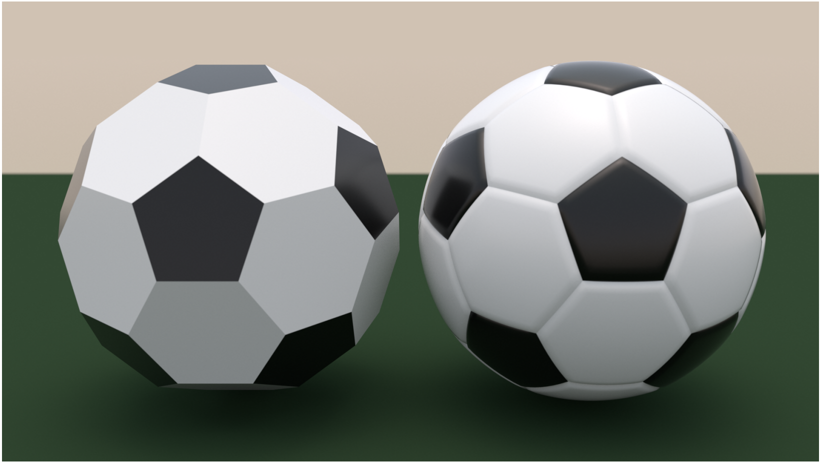

$\star 532$ - Picture from Wikipedia

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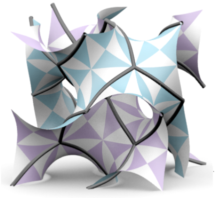

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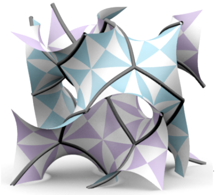

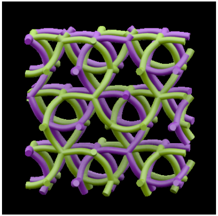

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- Only works for tame embeddings of graphs in $\mathbb{R}^3$.

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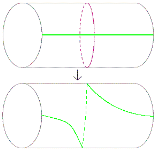
- Prime example: Dehn twist of green curve around red curve.



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  $\rightarrow$ computational group theory and algebra.

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- Implicit here is the description of Teichmüller space as equivalence classes of tilings, mod base 'point pushes' and hyperbolic isometries.

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- Orbifold group elements can be treated as closed curves $\rightarrow$ study the MCG of orbifolds by its action on simple closed curves.

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- Algebra is easier than geometry.

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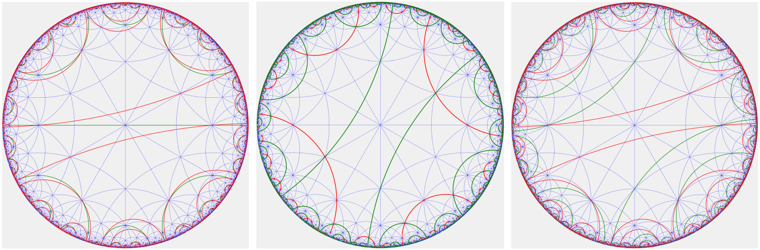

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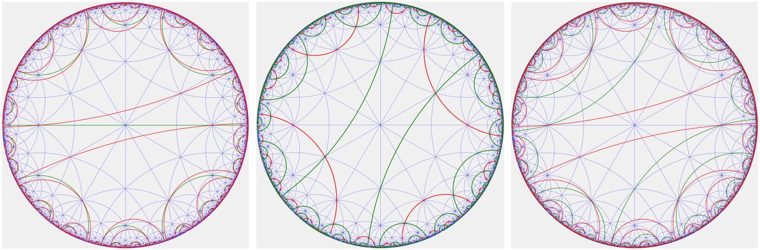

Figure: Hyperbolic Tilings that are related by elements of the mapping class group. The blue lines are used in the construction, the tiling is defined by only the green and red lines.

Examples of different tilings of the hyperbolic plane with the same combinatorial structure

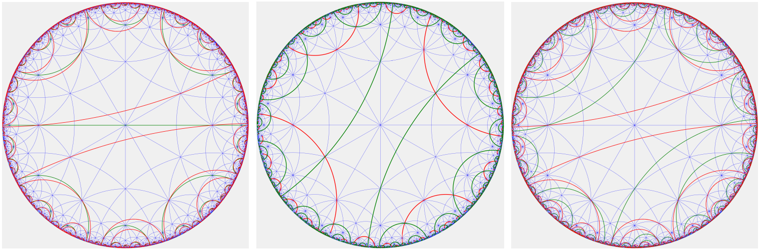

Figure: Hyperbolic Tilings that are related by elements of the mapping class group. The blue lines are used in the construction, the tiling is defined by only the green and red lines.

- Note that classical tiling theory does not treat these tilings because the tiles are unbounded.

Example of a Tiling of the Hyperbolic Plane and the Resulting Net

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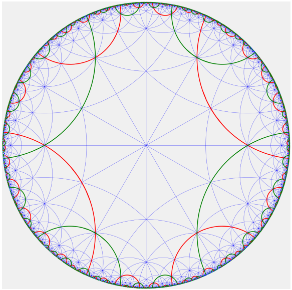

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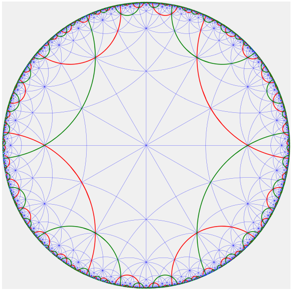

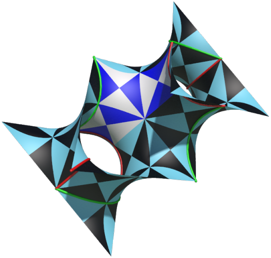

Figure: Hyperbolic Tiling and the corresponding drawing on the diamond surface in $\mathbb{R}^3$.

Example of a Tiling of the Hyperbolic Plane and the Resulting Net

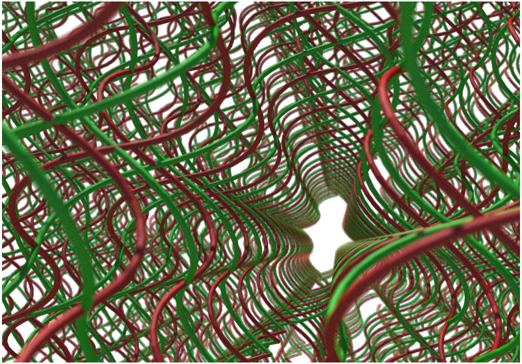

Figure: The corresponding net in $\mathbb{R}^3$, representing a molecular structure grown on the diamond surface with two distinct strands.

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- By using nice surfaces, one can study structures by examining them on the surface.
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- Because two-dimensions are nice, all (sufficiently simple) patterns with a given combinatorial structure can be produced from a single such pattern.
- The resulting structures have a natural ordering by complexity.
- Potential uses include systemically checking structures for certain physical properties, for possible synthetic materials.

Final Take-Home Message

Thank you for your Attention

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Work done in collaboration with
Myfanwy Evans, TUB; Vanessa Robins and Stephen Hyde, ANU