Space-optimal collaborative exploration of undirected graphs

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Berlin Mathematical School

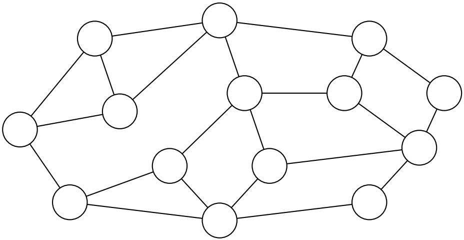
SPP 1736

Introduction and model

The octagonal Jubilee Maze at Symonds Yat - NotFromUtrecht - CC 3.0

Introduction and model

- k agents in undirected, initially unknown graph



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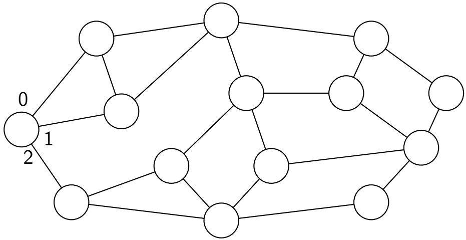

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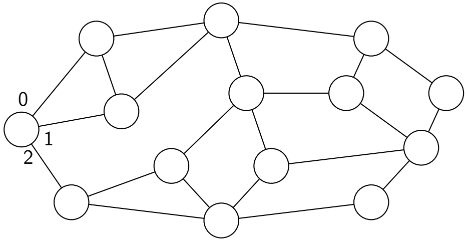

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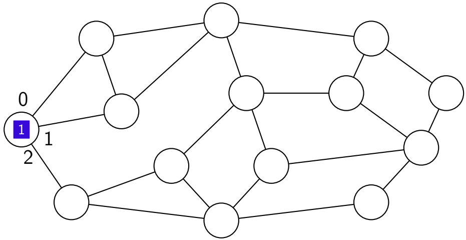

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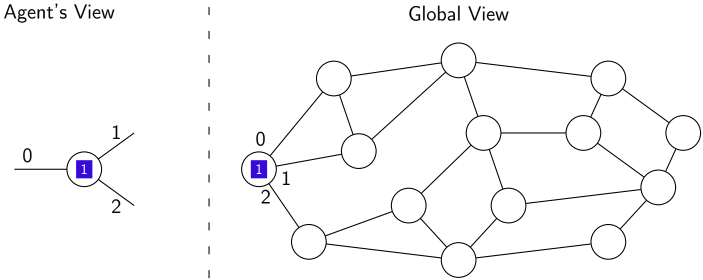

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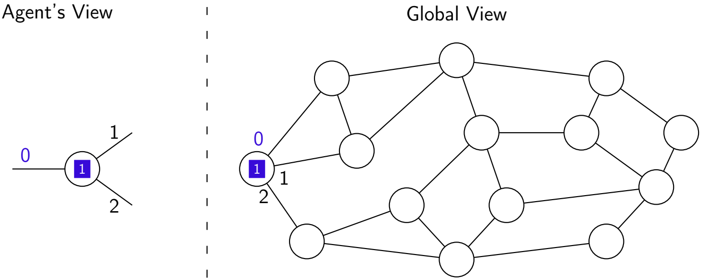

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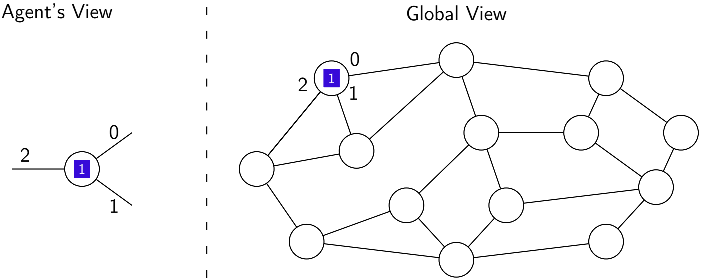

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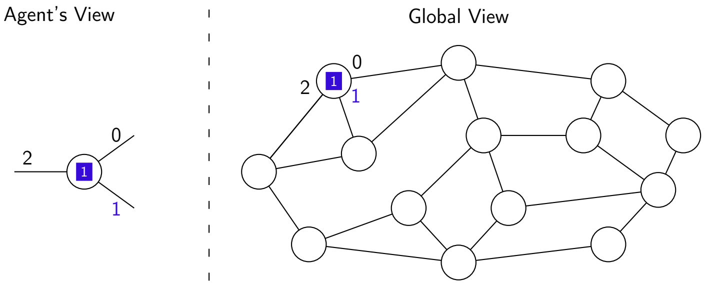

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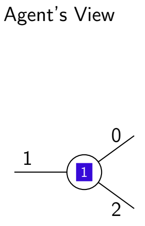

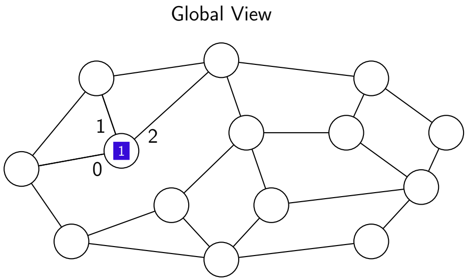

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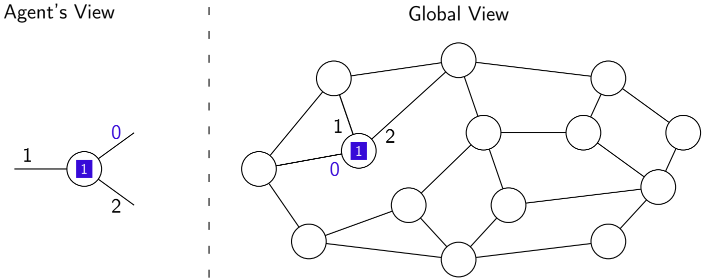

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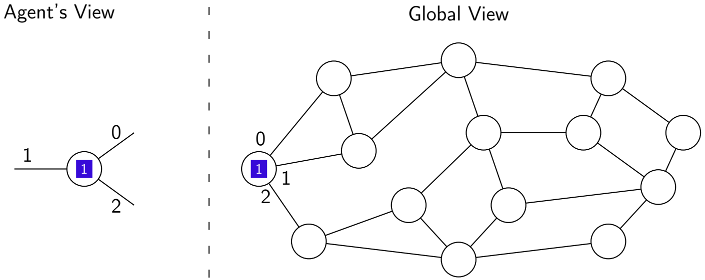

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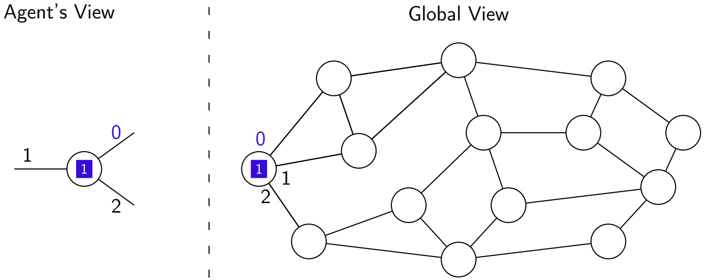

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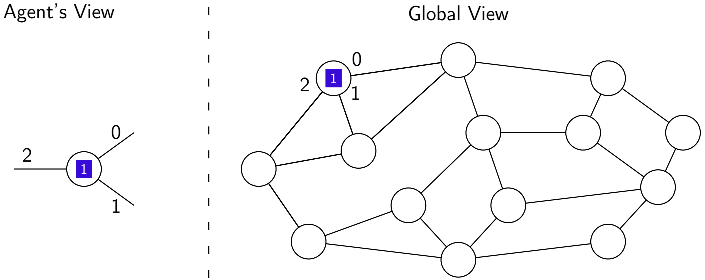

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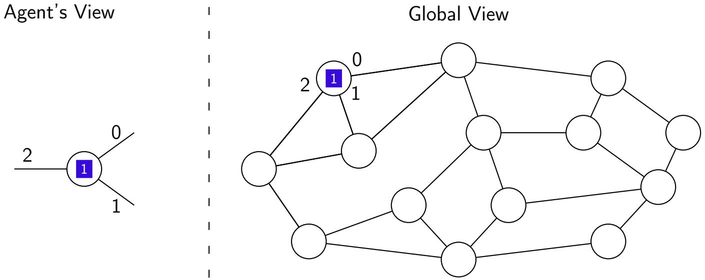

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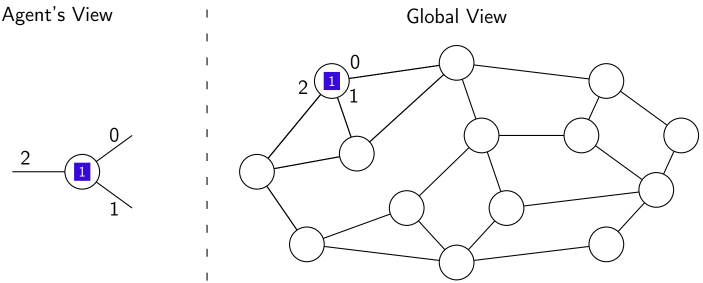

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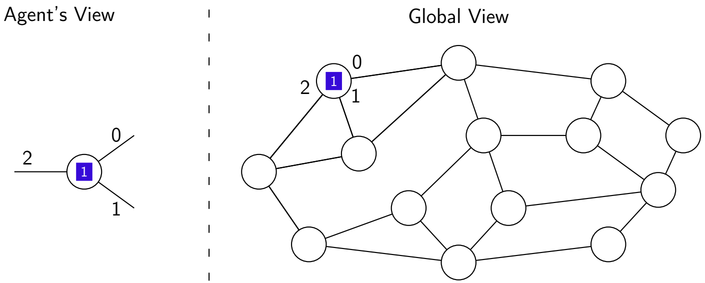

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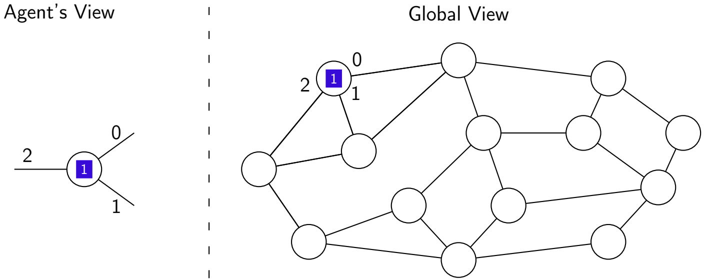

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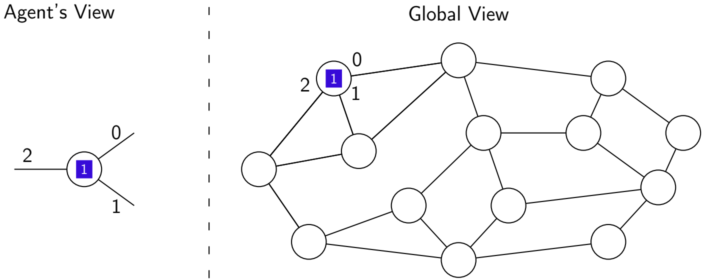

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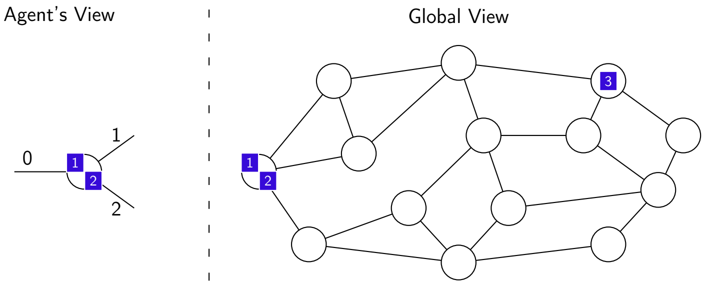

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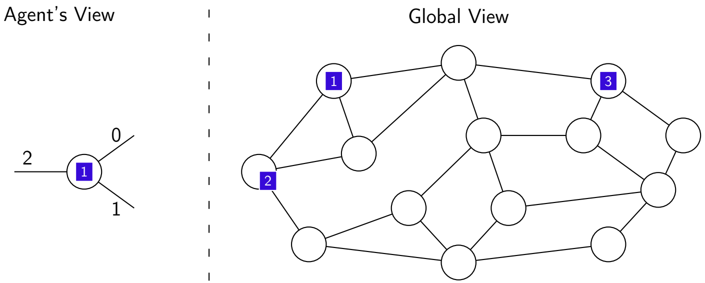

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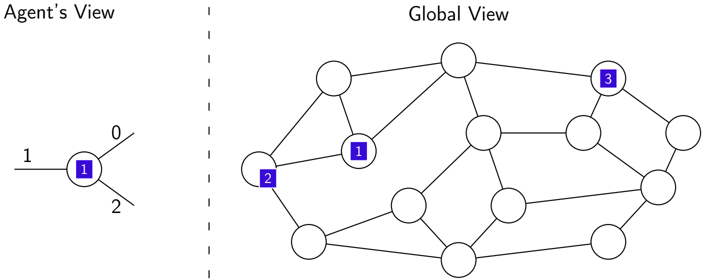

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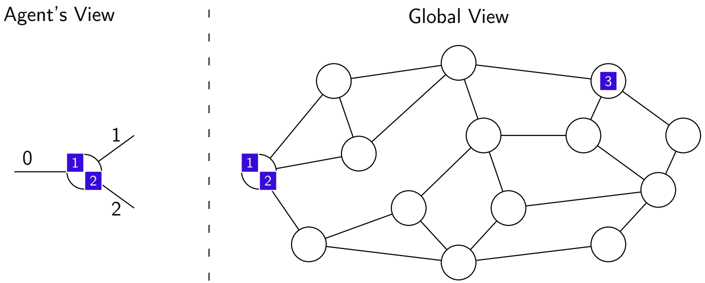

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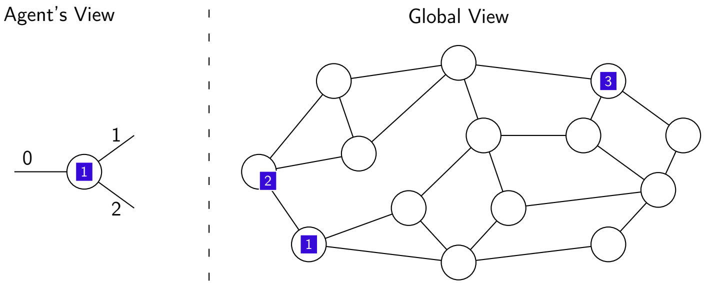

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Upper bound

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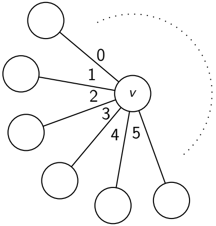

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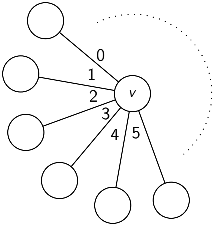

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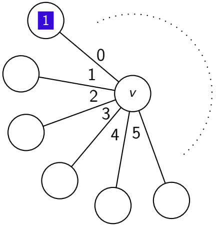

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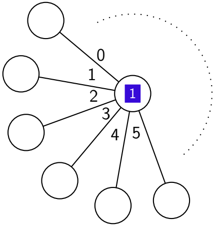

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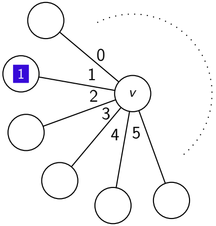

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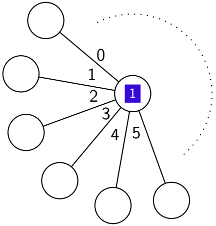

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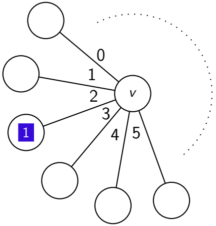

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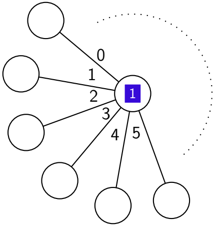

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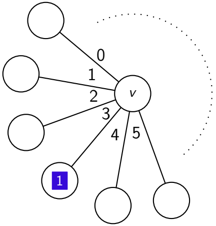

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$\Theta(\log\log n)$ agents

1 2 3 4 5 ....

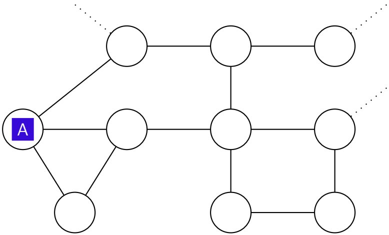
half of the agents move together

use half of the agents as markers

1 agent with $\Theta(\log\log n)$ bits

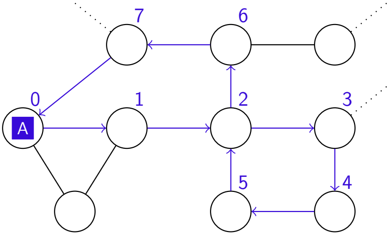
$\Theta(\log\log n)$ pebbles

Exploration algorithm - main idea



Exploration algorithm - main idea

- assume: agent A can find closed walk ω



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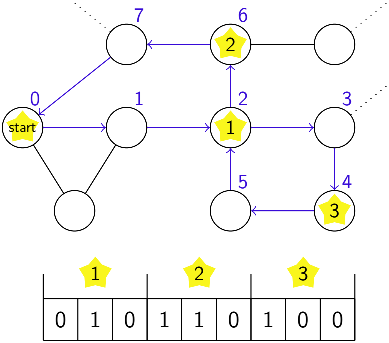

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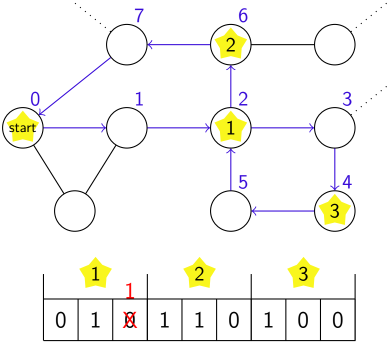

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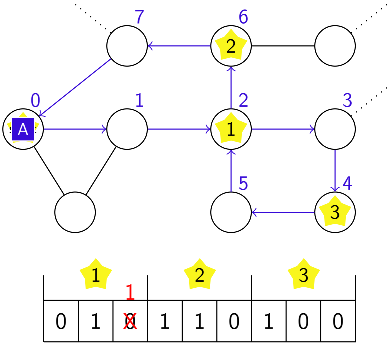

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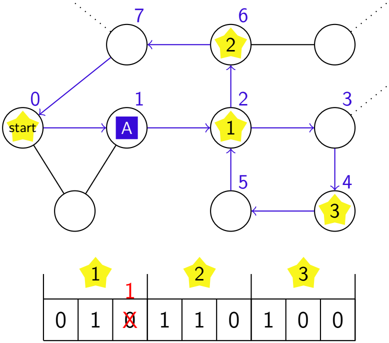

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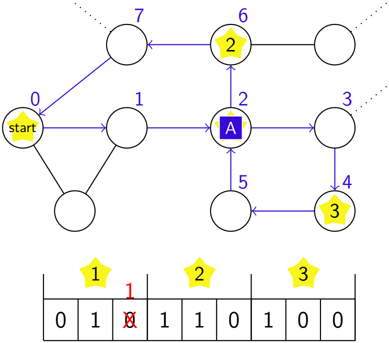

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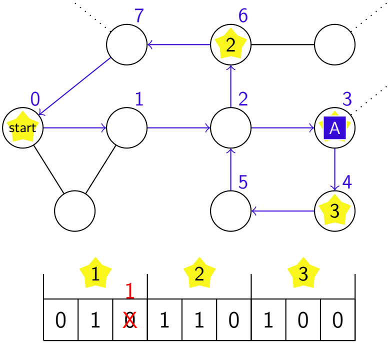

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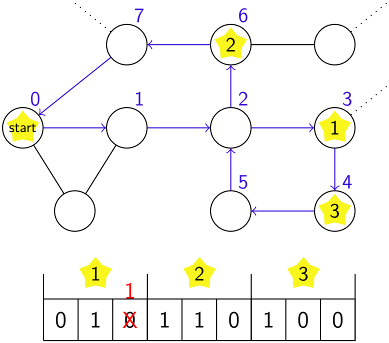

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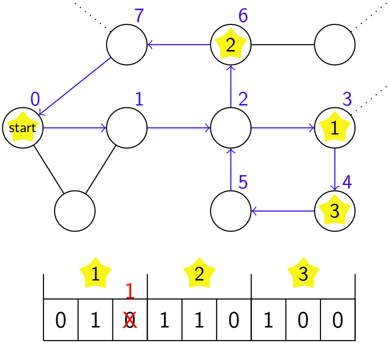

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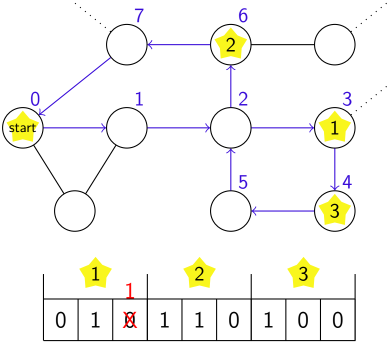

Challenges:

- vertices appearing multiple times along ω
- carrying the memory along while agent traverses graph

Exploration algorithm

Theorem (Reingold, JACM '08)

Agent with $\mathcal{O}(\log n)$ bits can explore any graph with n vertices.

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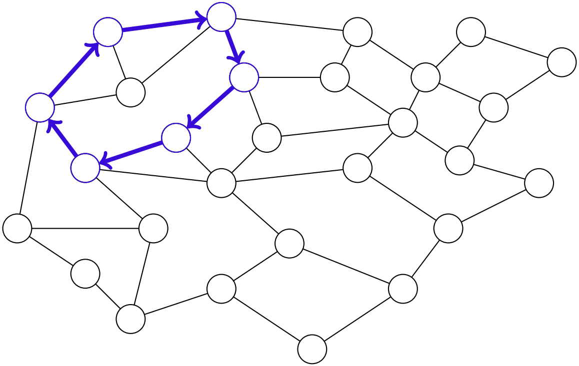

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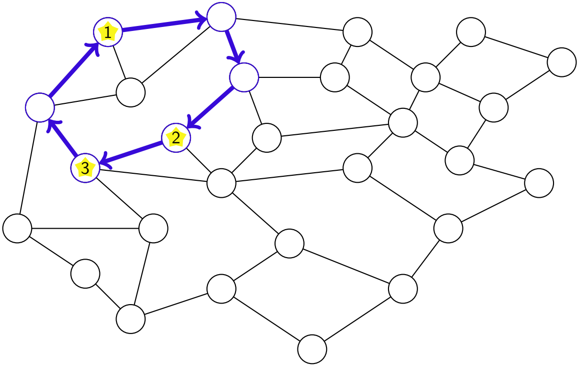

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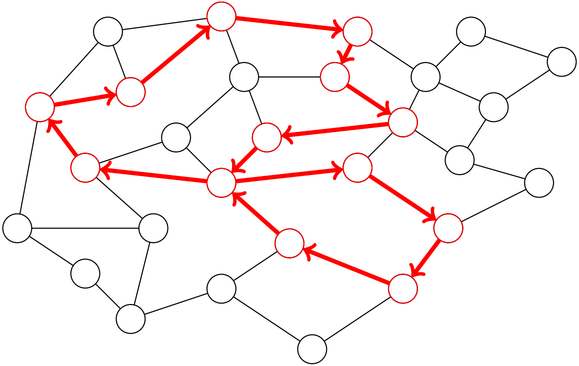

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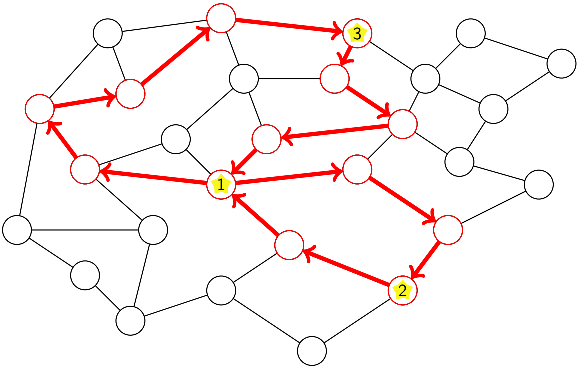

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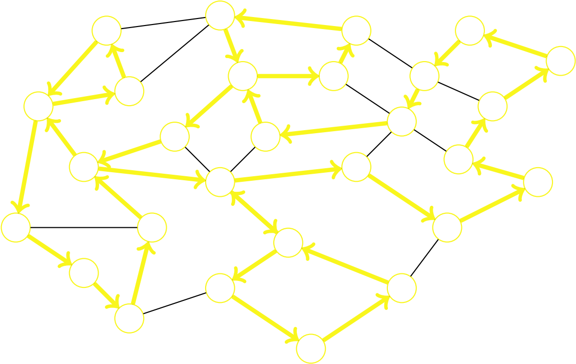

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Algorithm:

memory = constant # bits

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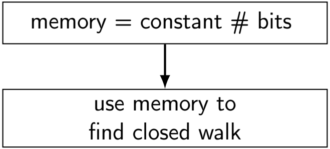

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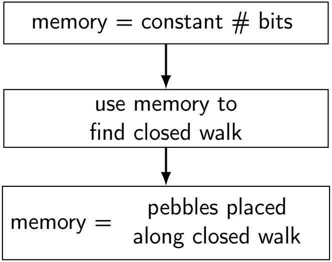

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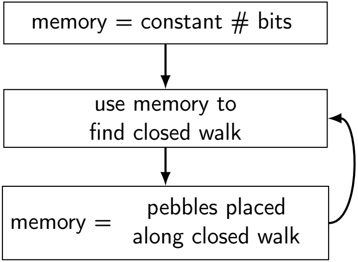

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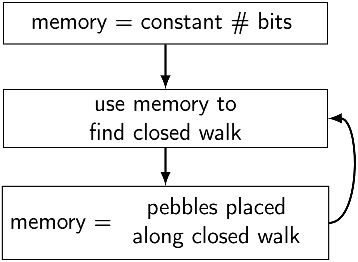

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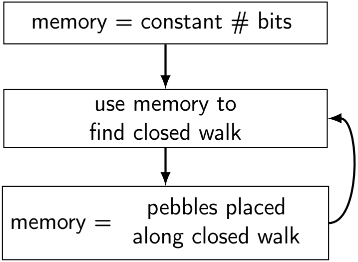

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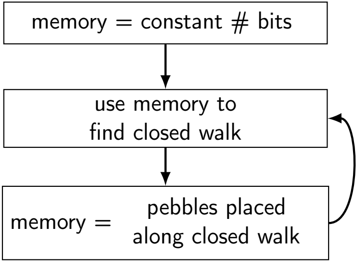

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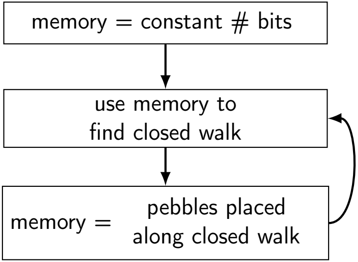

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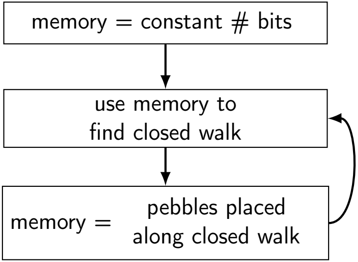

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   $\Rightarrow$ $\mathcal{O}(\log\log n)$ pebbles needed

Lower bound

Lower bound - building block

$\mathcal{A} =$ set of k agents with s states each

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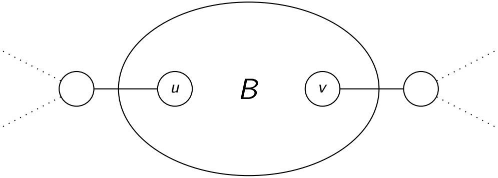
Definition (Barrier)

B is ***r*-barrier** for $\mathcal{A} \Leftrightarrow$ no subset of $\mathcal{A}$ of at most r agents can traverse B from u to v (or vice versa)



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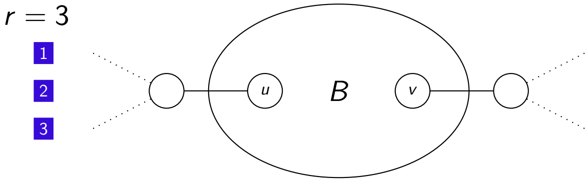

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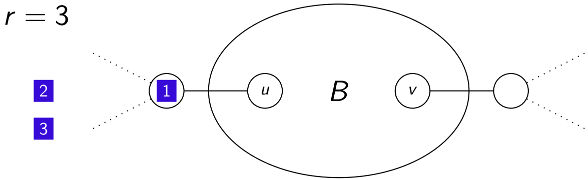

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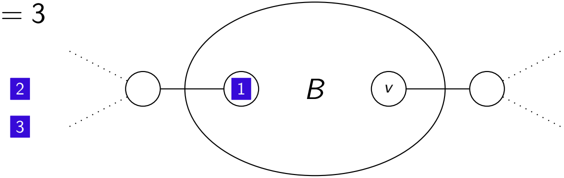

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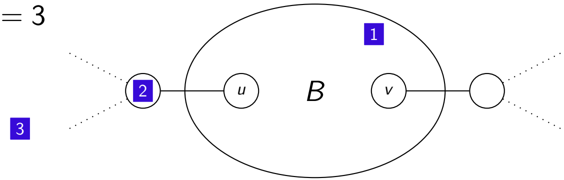

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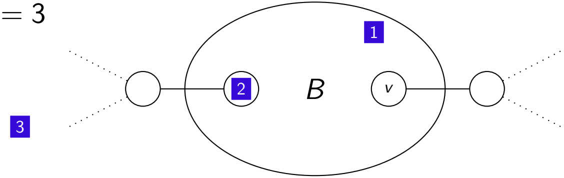

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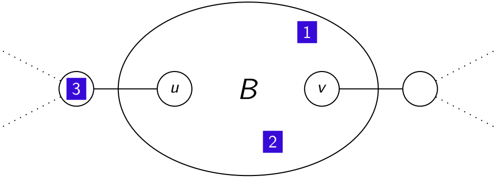

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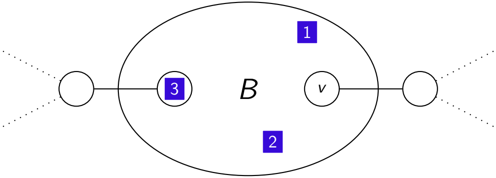

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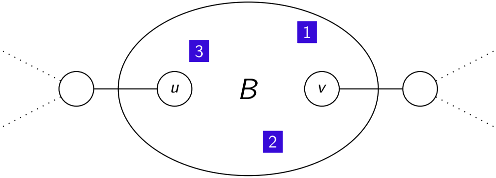

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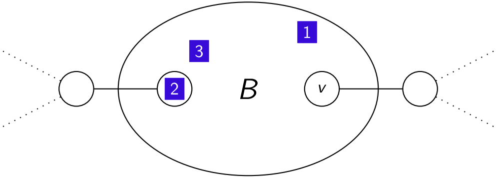

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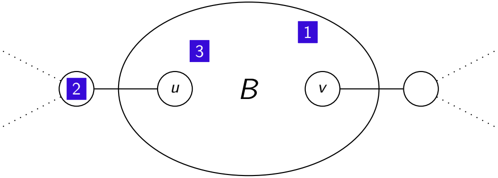

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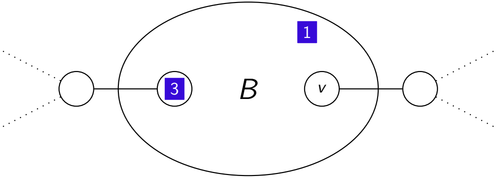

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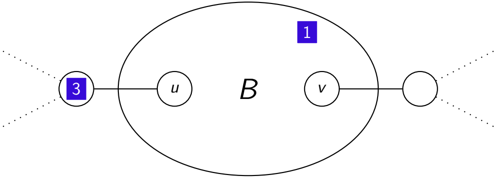

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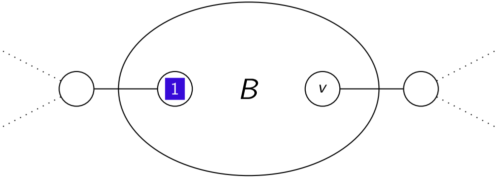

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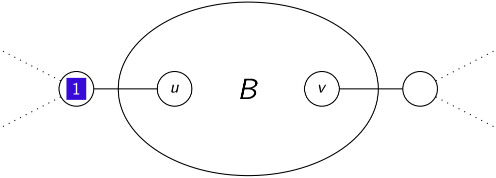

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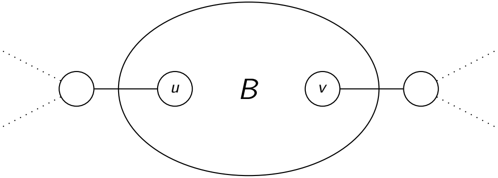

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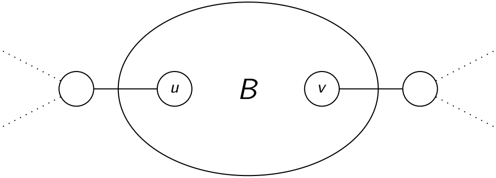

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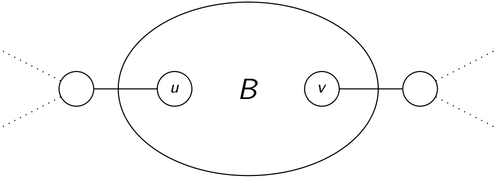

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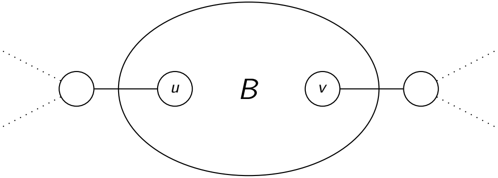

Observation

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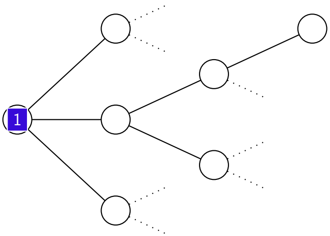
- Size of k-barrier $\Rightarrow$ lower bound on # agents and states per agent

Construction of 1-barrier [Fraigniaud et al., TCS '06]

- consider 1 agent in infinite 3-regular tree



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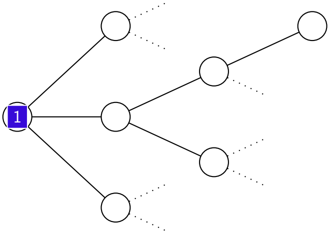

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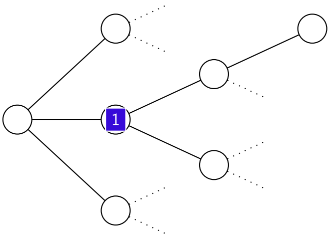

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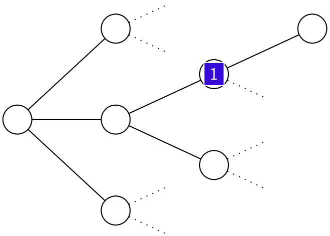

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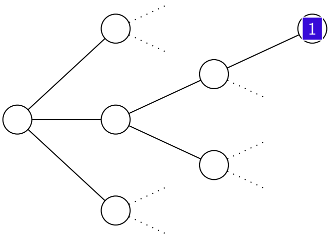

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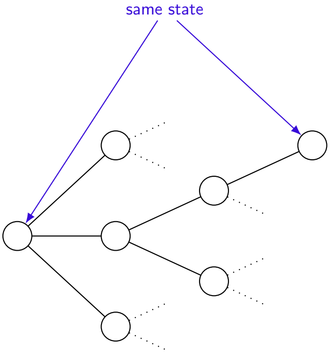

Construction of 1-barrier [Fraigniaud et al., TCS '06]

- consider 1 agent in infinite 3-regular tree
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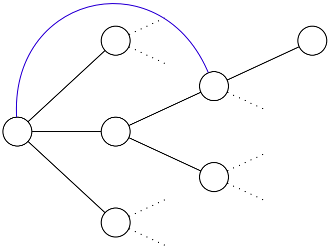

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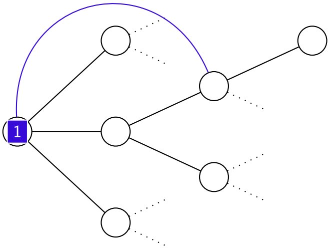

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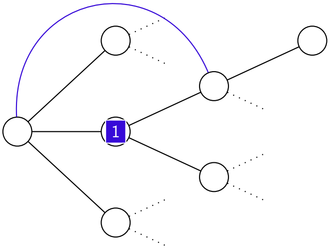

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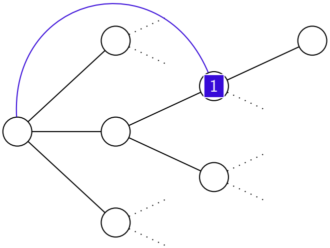

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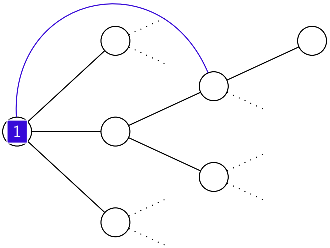

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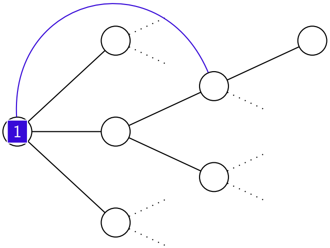

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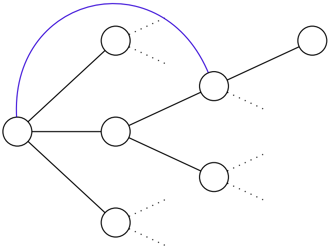

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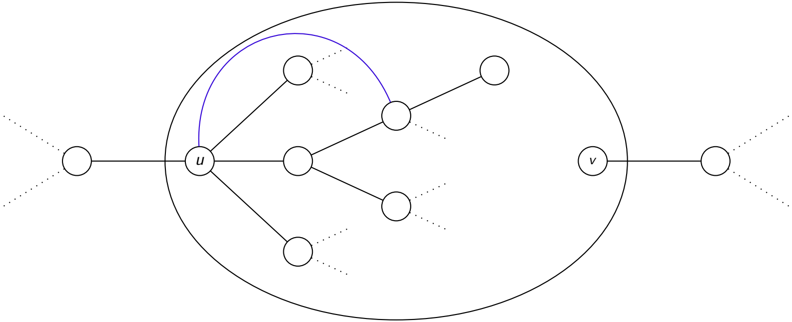

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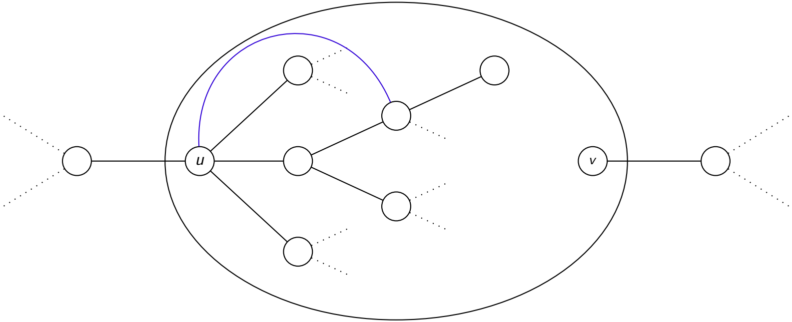

Lower bound - recursive construction

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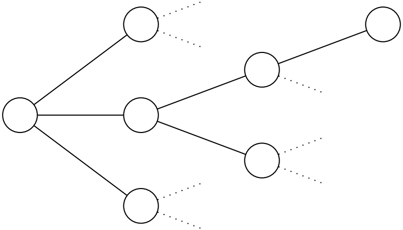

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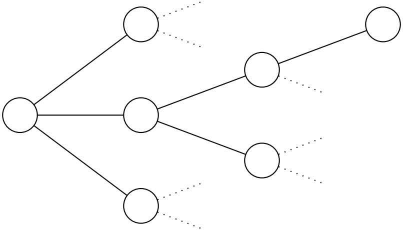

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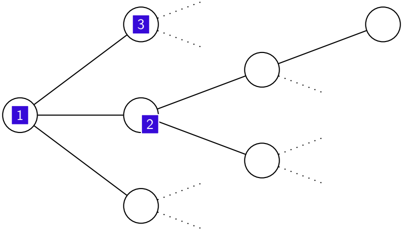

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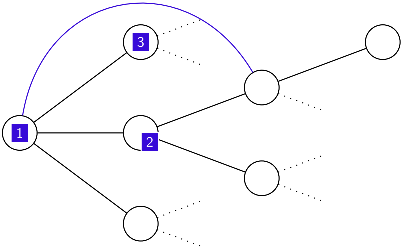

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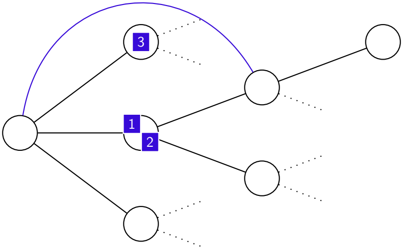

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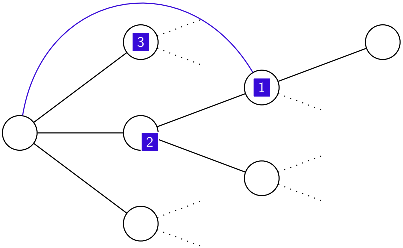

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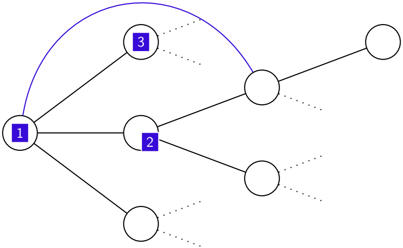

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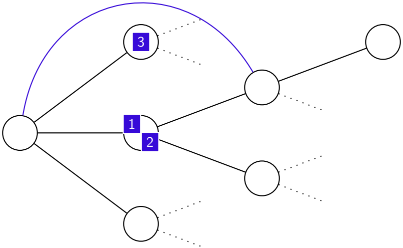

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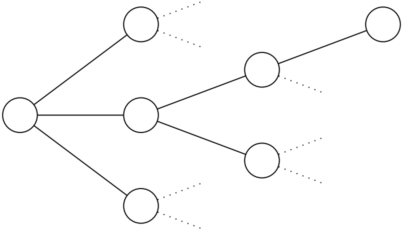

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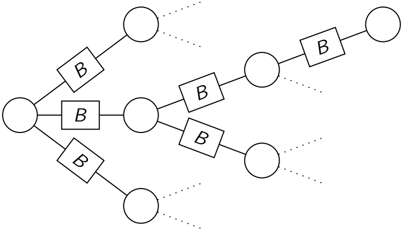

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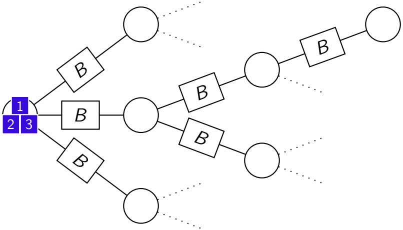

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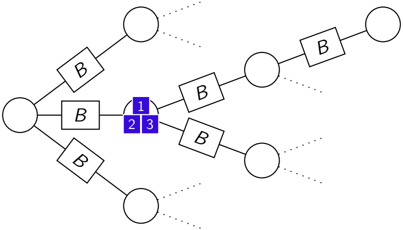

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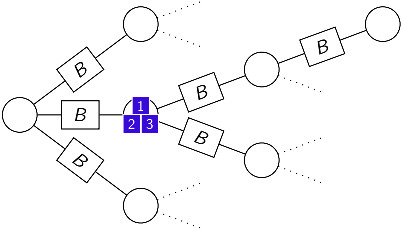

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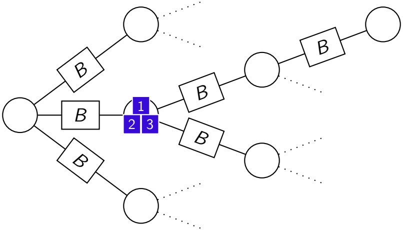

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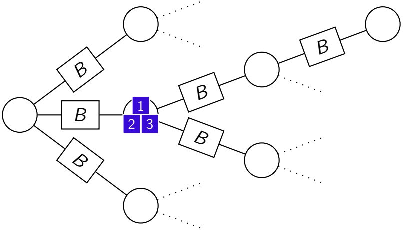

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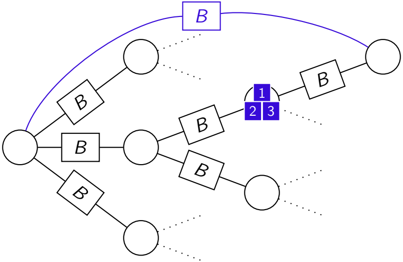

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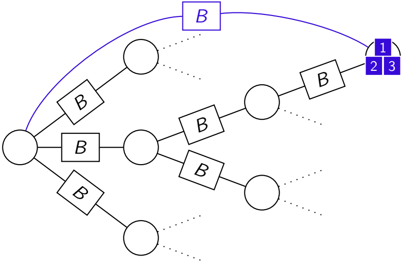

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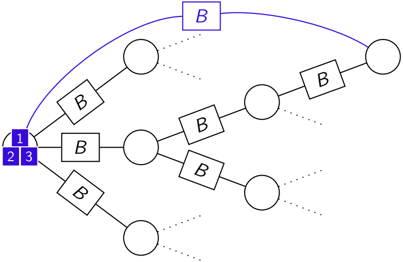

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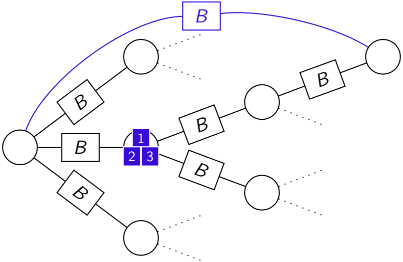

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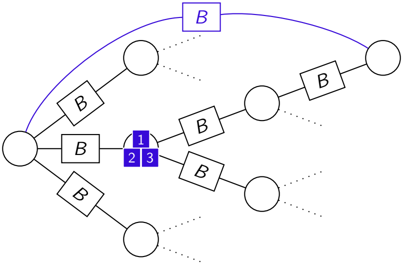

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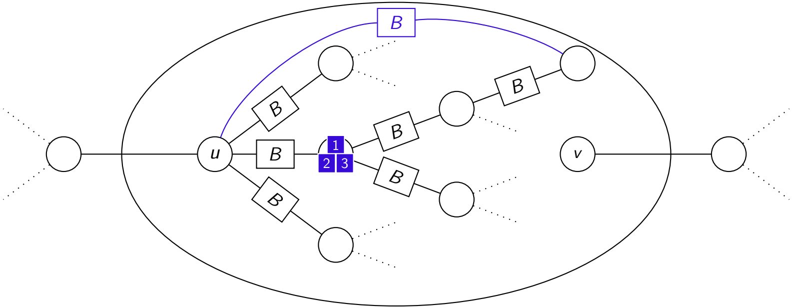

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Theorem (Disser, H., Klimm)

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$\Omega(\log\log n)$ *agents are necessary to explore any n vertex graph, if each agent has $\mathcal{O}((\log n)^{1-\varepsilon})$ bits of memory for $\varepsilon > 0$.*

Summary

Exploration of any graph with n vertices by 1 agent requires

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Thank you!